## BRITISH MATHEMATICAL OLYMPIAD

Round 2: Thursday, 15 February 1996

Time allowed Three and a half hours.

clearly marked.

Each question is worth 10 marks.

- Instructions Full written solutions not just answers are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt. Rough work should be handed in, but should be
  - One or two complete solutions will gain far more credit than partial attempts at all four problems.
  - The use of rulers and compasses is allowed, but calculators and protractors are forbidden.
  - Staple all the pages neatly together in the top left hand corner, with questions 1,2,3,4 in order, and the cover sheet at the front.

In early March, twenty students will be invited to attend the training session to be held at Trinity College, Cambridge (28–31 March). On the final morning of the training session, students sit a paper with just 3 Olympiad-style problems. The UK Team - six members plus one reserve - for this summer's International Mathematical Olympiad (to be held in New Delhi, India, 7–17 July) will be chosen immediately thereafter. Those selected will be expected to participate in further correspondence work between April and July, and to attend a short residential session 30 June–4 July before leaving for India.

Do not turn over until told to do so.

## BRITISH MATHEMATICAL OLYMPIAD

1. Determine all sets of non-negative integers x, y and z which satisfy the equation

 $2^x + 3^y = z^2.$ 

2. The sides a, b, c and u, v, w of two triangles ABC and UVW are related by the equations

 $u(v + w - u) = a^{2},$   $v(w + u - v) = b^{2},$  $w(u + v - w) = c^{2}.$ 

Prove that triangle ABC is acute-angled and express the angles U, V, W in terms of A, B, C.

3. Two circles  $S_1$  and  $S_2$  touch each other externally at K; they also touch a circle S internally at  $A_1$  and  $A_2$  respectively. Let P be one point of intersection of S with the common tangent to  $S_1$  and  $S_2$  at K. The line  $PA_1$  meets  $S_1$  again at  $B_1$ , and  $PA_2$  meets  $S_2$  again at  $B_2$ . Prove that  $B_1B_2$  is a common tangent to  $S_1$  and  $S_2$ .

4. Let a, b, c and d be positive real numbers such that

a+b+c+d = 12

and

$$abcd = 27 + ab + ac + ad + bc + bd + cd$$

Find all possible values of a, b, c, d satisfying these equations.